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Gravity Multiplet on KS and BB backgrounds

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Abstract

In this paper we study the spectra of glueballs on the Klebanov-Strassler background and its extension to the baryonic branch. We numerically calculate the mass spectrum of glueballs from the spin 2 “gravity” multiplet, which contains the traceless part of the stress-energy tensor and the transverse part of the $U(1)$ \mathcal{R} -current. The mass spectra of the corresponding fluctuations in supergravity coincide due to supersymmetry, which is manifest in the effective five-dimensional theory through a Supersymmetric Quantum Mechanics transformation. We show that the glueball spectra grow as $m_n^2 \propto Un^2$ for large values of the baryonic branch parameter U .

1 Introduction

Ideas of holography underlaid the AdS/CFT correspondence [1] provide a promising perspective for the study of gauge theories via string theory and supergravity. In particular, the extension of the holographic principle to a non-conformal case enables to capture the strongly coupled dynamics of a gauge theory through the classical supergravity. This allows, at least in principle, to calculate various correlation functions, extracting the masses of glueballs, which is not possible by means of the standard field theory technique. In this paper we focus on a particular case of the Klebanov-Strassler supergravity background [2] and its extension to the baryonic branch [3, 4, 5], which is a gravity dual of the non-conformal $\mathcal{N} = 1$ gauge theory with two pairs of matter multiplets.

According to the AdS/CFT correspondence, gauge theory operators correspond to fluctuations of the background supergravity fields. Thus the stress-energy tensor corresponds to the fluctuation of the metric. The transverse traceless part of the former combines with the transverse part of the $U(1)_{\mathcal{R}}$ current J_5^μ and the transverse fermionic superconformal current into a spin 2 massive supermultiplet [6]. The mass spectra of the corresponding glueballs coincide, what is evident from the supersymmetric structure of the equations of motion in the gravity dual theory.

In the gauge theory the supersymmetric structure of the spin 2 multiplet is transparent through the on-shell equation of current conservation [6]. Using superfield notations, one has

$$D^\alpha V_{\alpha\dot{\alpha}} = \bar{D}_{\dot{\alpha}} \bar{S}, \quad (1)$$

where $V_{\alpha\dot{\alpha}} = \sigma_{\alpha\dot{\alpha}}^\mu V_\mu$ is a real supercurrent that contains the $T^{\mu\nu}$ and the J_5^μ current,

$$V^\mu = J_5^\mu - \frac{i}{2} \theta \sigma^\nu \bar{\theta} T_\nu{}^\mu + i \theta^2 \partial^\mu \bar{s} - i \bar{\theta}^2 \partial^\mu s + \frac{1}{4} \bar{\theta}^2 \theta^2 (2D^\mu + \square J_5^\mu) + \text{fermions}. \quad (2)$$

The supercurrent V^μ contains two supermultiplets: the transverse spin-2 multiplet, which consists of the traceless transverse components of $T^{\mu\nu}$ and the transverse part of J_5^μ , and the chiral multiplet S containing the trace $T_\mu{}^\mu$, the divergence $\partial_\mu J_5^\mu$ and the γ -trace of the superconformal current. It also contains the complex scalar field s as its lowest component. The chiral multiplet S thus accounts for the anomalies of the scale, $U(1)_{\mathcal{R}}$ and the superconformal symmetries, associated with the components of the supercurrent, while the equation (1) is the supersymmetric generalization of the anomalous divergence of the current.

On the gravity dual side, background fluctuations of supergravity fields that are dual to the operators in (2) are massless in the five-dimensional sense if the theory is conformal. If the conformal symmetry is broken, the dual of the $U(1)_{\mathcal{R}}$ current satisfies the equation for a massive vector particle in five dimensions. Note that the dual of the traceless transverse part of the stress-energy tensor is described by a five-dimensional massless equations in both cases. However the finite warp-factor at the tip of the conifold in the non-conformal case leads to a finite four-dimensional spectrum of glueballs.

In this work we describe the holographic dual modes of the traceless part of (2), namely the spin 2 *gravity* multiplet, in the context of the baryonic branch of the KS

background. The baryonic branch is a continuous family of the type IIB supergravity solutions originating at the KS background. The branch is parameterized by the vevs of the baryonic operators in the dual gauge theory [3, 7]. The backgrounds from the family are constructed in terms of the Papadopoulos and Tseytlin [8] ansatz, which consists of scalar functions parameterizing the metric and fluxes. Those scalar functions depend only on the radial coordinate of the conifold t and satisfy a system of first order differential equations, which was derived in [4]. No analytical solution to this system is known, except for the KS case¹, and in practice the backgrounds from the baryonic branch have to be constructed numerically. More details about the baryonic branch solutions can be found in the works [4, 5].

The KS and baryonic branch backgrounds correspond to non-conformal gauge theories. We derive the linearized equations for the vector fluctuation dual to the $U(1)$ \mathcal{R} -current and the metric fluctuation dual to the stress-energy tensor. Then we numerically compute the four-dimensional mass spectra of the corresponding glueballs along the branch. Since we derive our equations on the solid ground of “microscopic” ten-dimensional theory, we can test the applicability of the similar results obtained in [10] through an effective approach of five-dimensional models of gauge/gravity correspondence.

This paper is organized as follows. In the section 2 we remind the reader the dual description of the bosonic operators of the gravity and anomaly multiplets in the case of the KS background. This part also contains a sketch of the holographic anomaly mechanism suggested in [11].

Sections 3 and 4 are dedicated to a derivation of the linearized equations for the bosonic fluctuations of fields dual to the gravity multiplet of (2). On the supergravity side the bosonic sector of the multiplet consists of the symmetric traceless perturbation of the metric – the graviton, and the transverse vector perturbation discussed in the section 2.

Since the bosonic fluctuations of the gravity multiplet are related by supersymmetry, their spectra coincide. There is a Supersymmetric Quantum Mechanics (SQM) transformation relating the effective five-dimensional equations, which is a reminiscence of the original supergravity transformation in ten dimensions. In section 4 we derive the equation for the vector mode only for the case of the KS background, but the supersymmetric structure of the equations allows us to extend it further to the baryonic branch. We show that this equation is the same as discovered by [10] in the five-dimensional approach. This is discussed in detail in section 5.

We present the results of a numerical calculation of the spectrum in the section 6. Although the equations that describe the graviton and the vector particle yield the same spectrum of bound states, they are essentially different. We perform two separate calculations of the spectrum of the gravity multiplet which is an important consistency check of the numerical results. We conclude with a discussion in section 7.

¹The analytical solution [9], known as Maldacena-Nunez background, also solves the system of [4]. However, it has different boundary conditions at infinity and therefore does not belong to the baryonic branch.

2 Multiplets and Anomalies in the Dual Theory

The purpose of this work is the study of the gravity multiplet, i.e. the fluctuations above a classical supergravity background dual to a field theory supermultiplet consisting of the traceless part of the stress-energy tensor $T^{\mu\nu}$, spin 3/2 conformal supercurrent and the conserved part of the $U(1)_{\mathcal{R}}$ current J_5^μ . The gravity multiplet therefore contains the traceless symmetric excitation of the metric – the graviton $h_{\mu\nu}$, the spin 3/2 gravitino and the transverse vector excitation \tilde{A}_μ along the 1-form $d\psi$ which we specify below. Classical supergravity backgrounds that we study in this work are the backgrounds from the baryonic branch of the KS solution.

First we consider the transverse non-diagonal fluctuation of the background metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (3)$$

where $h_{\mu\nu}$ has only components in the Minkowskian directions. We find that in accordance with the general results of [12, 13] this excitation is described by the massless scalar minimally coupled to the metric in the Einstein frame for all backgrounds of the baryonic branch.

Next we consider fluctuation that is dual to the conserved (transverse) part of the $U(1)_{\mathcal{R}}$ current in the KS background. Recall that the KS solution corresponds to a manifold that is locally a product of the Minkowski space-time and the six dimensional deformed conifold [2],

$$ds_{\text{KS}}^2 = h^{-1/2}(t)\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(t)ds_6^2, \quad (4)$$

where $h(t)$ is the warp factor that depends on the radial coordinate t of the conifold, related to the standard conical radial coordinate r via $t \sim 3 \log r$. $U(1)_{\mathcal{R}}$ transformations act as rotations along the conifold base $T^{1,1}$,

$$\psi \rightarrow \psi + \zeta,$$

where ψ is one of the angles on the base.²

In the conformal case, the background is invariant under this symmetry, what results in a massless gauge field \tilde{A}_μ . The KS background, as well as the backgrounds along the baryonic branch, breaks the $U(1)_{\mathcal{R}}$ symmetry already in the UV. The 2-form potential for the RR form F_3 has an explicit ψ dependence. In the UV limit

$$C_2 \simeq M\psi\omega_2,$$

where M is the flux of F_3 through the S^3 of $T^{1,1}$, and ω_2 is the ψ independent 2-form on $T^{1,1}$. Given that ψ itself is a double cover of the circle, C_2 breaks $U(1)_{\mathcal{R}}$ down to \mathbb{Z}_{2M} in the UV. In the IR the metric has an explicit ψ dependence that breaks \mathbb{Z}_{2M} further to \mathbb{Z}_2 in the full agreement with the gauge theory.

²One can think of $T^{1,1}$ as of the space $S^3 \times S^3/U(1) \simeq S^3 \times S^2$. The angle ψ is obtained by the identification of the 3rd Euler angles of both S^3 .

As a result, the corresponding fluctuation of the background acquires mass that is not vanishing even in the UV region [11, 14]. The fluctuation in question modifies the metric along the ψ direction $g_{\mu\psi}$ and can be described by the perturbation of the 1-form $d\psi$ by the “gauge” field $\tilde{A} = \tilde{A}_\mu dx^\mu + \tilde{A}_t dt$,

$$d\psi \rightarrow d\psi + \tilde{A}. \quad (5)$$

Since the dependence on the angles of the conifold is not important, we can restrict our attention to the five-dimensional theory. In the conformal case, in the absence of the 3-form fluxes, the five-dimensional vector field \tilde{A} satisfies the equation for the massless vector

$$d * _5 d\tilde{A} = 0. \quad (6)$$

The longitudinal part of \tilde{A} is not fixed by the equation (6) as it is a gauge degree of freedom. The corresponding symmetry is anomaly free. After adding the fluxes, the equation for \tilde{A} can be brought to the form

$$d(f * _5 d(g\tilde{A})) + * _5 \tilde{A} = 0, \quad (7)$$

with some background-dependent functions f and g . The longitudinal part of \tilde{A} is no longer trivial and satisfies

$$d * _5 \tilde{A} = 0. \quad (8)$$

For an observer in four dimensions, the five-dimensional no-source equation (8) is precisely the equation with an anomalous source

$$\partial^\mu \tilde{A}_\mu = \theta(\Lambda), \quad (9)$$

where μ denotes the space-time indices. This holographic anomaly mechanism is discussed in more detail in [11].

The backgrounds we are interested in have a global $SU(2) \times SU(2)$ symmetry. Since we are interested in the uncharged sector, all fluctuations should be s -waves with respect to the directions along the base of the conifold. This is obvious for the four-dimensional metric fluctuations as we keep it angle-independent. In the case of the vector, it is more tricky. In fact we need to switch from the 1-form $d\psi$ to the invariant extension $g^5 \rightarrow g^5 + \tilde{A}$, where

$$g^5 = d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2. \quad (10)$$

Apparently the shift of $d\psi$ results in the same shift of g^5 .

The anomalies of the scale, superconformal and $U(1)_{\mathcal{R}}$ symmetries form a chiral supermultiplet [6]. Its bosonic part on the gravity side contains the fluctuations of the metric trace $h_\mu{}^\mu$ and the longitudinal part of the vector field $\tilde{A}_\mu = \partial_\mu \tilde{a}$.

In the section 4 we derive the equation (7) for the transverse part of the vector fluctuation (5). The transverse component decouples from the longitudinal part and from other supergravity fluctuations. Unfortunately it is much more complicated to derive the equation for the longitudinal mode \tilde{a} . Moreover there are certain indications that it does

not decouple from the other fields and needs to be considered as a part of a more complicated system [15]. Coupling with different supergravity excitations will lead to some non-trivial right hand side of the equation (9). It is particularly interesting to find the supergravity expression for $\theta(\Lambda)$ and compare it with the gauge theory predictions. This task is more ambitious and we leave it for a future work.

3 Graviton Equations

In the current and the following sections we will be interested in the equations for the bosonic components of the gravity multiplet, the graviton $h_{\mu\nu}$ and the vector mode \tilde{A}_μ . We start with a ten-dimensional analysis of the linearized supergravity equations for the graviton excitations, valid for any solution on the baryonic branch, and proceed with a derivation of the equations for the vector field in the KS background in the section 4.

The traceless symmetric perturbation of the metric is described by the five-dimensional Klein-Gordon equation for a minimal scalar coupled to the background in the Einstein frame. A straightforward check [16] shows that this property holds for the whole baryonic branch.

Here we follow the notations of Papadopoulos and Tseytlin [8]. The functions $A(t)$, $p(t)$, $x(t)$ and $\Phi(t)$ below are scalar functions from the PT ansatz depending on the radial variable t . In particular, $A(t)$ is equivalent to the warp factor in the KS case $e^{-2A} = h^{1/2}$. It should not be confused with the vector fluctuation of the metric $\tilde{A} = \tilde{A}_i dx^i$. In the Einstein frame the equation for the fluctuation of the graviton $\delta(ds^2) = e^{-2A} h_{\mu\nu} dx^\mu dx^\nu$ takes the form

$$\ddot{h}_{\mu\nu} + 2(\dot{x} - \dot{\Phi} + 2\dot{A})\dot{h}_{\mu\nu} - k^2 e^{-2A-6p-x} h_{\mu\nu} = 0, \quad (11)$$

where k^2 is the square of the 4-momentum and the over dots stand for t derivatives. The equation (11) is precisely the Klein-Gordon equation for the minimal scalar in the the baryonic branch backgrounds including the KS point.

To proceed to the explicit form of the equation (11) for the KS background one chooses

$$\begin{aligned} e^{-2A} &= h^{1/2}, & e^{6p+2x} &= \frac{3}{2} (\coth t - t \operatorname{csch}^2 t), \\ e^\Phi &= e^{\Phi_0} = 1, & e^{2x} &= \frac{1}{16} (\sinh t \cosh t - t)^{2/3} h, \end{aligned} \quad (12)$$

where $h(t)$ is the warp factor of the metric (4):

$$h(t) = \int_t^\infty dx \frac{(x \coth x - 1)(\sinh 2x - 2x)^{1/3}}{\sinh^2 x}. \quad (13)$$

With these assignments the equation takes the familiar form [17]

$$\ddot{h}_{\mu\nu} + \frac{8}{3} \frac{\sinh^2 t}{\sinh 2t - 2t} \dot{h}_{\mu\nu} - k^2 \frac{h(t) \sinh^2 t}{(\sinh 2t - 2t)^{2/3}} h_{\mu\nu} = 0. \quad (14)$$

In the last term we absorbed the numerical constants in the normalization of the momentum. It is also convenient to write the equation in the conventional Shroedinger form

$$(-\partial_t^2 + V_2(t))h_{\mu\nu} = 0, \quad (15)$$

with the effective potential $V_2(k^2, t)$ given by

$$V_2 = \frac{k^2 h(t) \sinh^2 t}{(\sinh 2t - 2t)^{2/3}} - \frac{8}{9} \frac{\sinh^4 t}{(\sinh t \cosh t - t)^2} + \frac{4}{3} \frac{\sinh t \cosh t}{(\sinh t \cosh t - t)}. \quad (16)$$

4 Vector Mode

To find a supergravity excitation that corresponds to the $U(1)_{\mathcal{R}}$ current J_5^μ , one should consider a special deformation along the angular direction $\partial/\partial\psi$ of the $T^{1,1} \simeq S^3 \times S^2$, as was discussed in the section 2. We perturb the $SU(2) \times SU(2)$ invariant 1-form g^5 (10) in the following way:

$$g^5 \rightarrow g^5 + 2\tilde{\beta}(t)\tilde{A}, \quad \tilde{A} \equiv \tilde{A}_\mu dx^\mu, \quad (17)$$

where \tilde{A} is a 1-form describing the vector mode and $\tilde{\beta}(t)$ is yet unknown function of t . Such a deformation leads to the following perturbation of the metric:

$$ds^2 \rightarrow ds^2 + 2l(t)g^5 \cdot \tilde{A}, \quad (18)$$

where we introduced $l = 2\tilde{\beta}e^{-6p-x}$ for a latter convenience. This change of the metric will affect the Einstein equation as well as other equations of the type IIB supergravity. In particular one needs to modify the RR 5-form F_5 to preserve its self-duality:

$$\begin{aligned} \delta F_5 = & -\beta\tilde{A} \wedge dg^5 \wedge dg^5 + \beta d\tilde{A} \wedge g^5 \wedge dg^5 + \beta e^{3p+x/2} *_5 d\tilde{A} \wedge dg^5 + \\ & + 2e^{-2x}(\beta - \tilde{\beta}K)e^{-3p-x/2} *_5 \tilde{A} \wedge g^5. \end{aligned} \quad (19)$$

Here $\beta(t)$ is yet another function to be determined and $K = 4\dot{A}e^{2x}$ in the PT notations [8]. This turns out to be a minimal ansatz required for the KS solution. One can show that there is no need to perturb the other type IIB fields if one is interested only in the four-dimensional transverse part of \tilde{A} .

The ansatz so far contains the unknown functions β , and $\tilde{\beta}$ or l , which can be fixed by the equations of motion. The Bianchi identity provides us with the following equations:

$$d(\beta e^{3p+x/2} *_5 d\tilde{A}) + 2e^{-2x}(\beta - \tilde{\beta}K)e^{-3p-x/2} *_5 \tilde{A} = 0, \quad (20)$$

and a simple equation for the function β ,

$$\dot{\beta} = 0, \quad \text{or} \quad \beta = \beta_0. \quad (21)$$

To find the function $\tilde{\beta}(t)$, or $l(t)$, one should linearize the Einstein equation with the perturbation of the metric as in (18). The only nontrivial equation comes from the $\delta R_{\mu\psi}$ term. After certain simplifications one can write it in the form

$$\begin{aligned} \partial_t^2 \tilde{A}_\mu + \left(2(\dot{l}/l) + 6\dot{p} + 3\dot{x} + 2\dot{A} \right) \partial_t \tilde{A}_\mu - k^2 e^{-2A-6p-x} \tilde{A}_\mu + \\ + \left((\ddot{l}/l) + (\dot{l}/l)(6\dot{p} + 3\dot{x} + 2\dot{A}) - 2\dot{A}(6\dot{p} + \dot{x}) - 2e^{-12p-4x} \right) \tilde{A}_\mu = \\ = \left(\frac{e^{-6p-x}}{24} (H_3^2 + F_3^2) - \frac{2\beta_0}{l} e^{-6p-5x} K + \frac{1}{2} e^{-4x} K^2 \right) \tilde{A}_\mu. \end{aligned} \quad (22)$$

In the KS background the square of the 3-forms is given by

$$F_3^2 = H_3^2 = 3e^{6p-x} \frac{t^2 + 2t^2 \cosh^2 t - 6t \sinh t \cosh t + \cosh^2 t - 2 + \cosh^4 t}{\sinh^4 t}. \quad (23)$$

If one now writes the equation (20) in components, taking into account (21) and the transversality condition $\partial^\mu \tilde{A}_\mu = 0$,

$$\begin{aligned} \partial_t^2 \tilde{A}_\mu + (6\dot{p} + \dot{x} + 2\dot{A}) \partial_t \tilde{A}_\mu - k^2 e^{-2A-6p-x} \tilde{A}_\mu + \\ + \left(8\tilde{\beta} \dot{A} e^{-12p-2x} / \beta_0 - 2e^{-12p-4x} \right) \tilde{A}_\mu = 0, \end{aligned} \quad (24)$$

and compares it with the equation (22), one will find that two equations coincide only for

$$\beta_0 = 1, \quad \text{and} \quad l = e^{-x}. \quad (25)$$

Thus, the equation (24) with the solution (25) describes the transverse vector excitation of the KS supergravity solution. For computation of the mass spectrum it is worth writing (24) in terms of the explicit solution (12). We obtain the equation

$$\partial_t^2 \tilde{A}_\mu + \mathcal{P}(t) \partial_t \tilde{A}_\mu + \mathcal{Q}(t) \tilde{A}_\mu = 0, \quad (26)$$

with³

$$\mathcal{P}(t) = \frac{4}{3} \frac{\sinh^2 t}{(\sinh t \cosh t - t)} - 2 \coth t - \frac{\dot{h}}{h}, \quad (27)$$

$$\mathcal{Q}(t) = -\frac{k^2 h \sinh^2 t}{(\sinh 2t - 2t)^{2/3}} - \frac{8}{9} \frac{\sinh^4 t}{(\sinh t \cosh t - t)^2} - \frac{2}{3} \frac{\dot{h} \sinh^2 t}{(\sinh t \cosh t - t)h}. \quad (28)$$

Again, one could write the above equation in the form (15) with the new effective potential $V_1(k^2, t)$,

$$\begin{aligned} V_1 = \frac{1}{2} \dot{\mathcal{P}} + \frac{1}{4} \mathcal{P}^2 - \mathcal{Q} = \frac{k^2 h \sinh^2 t}{(\sinh 2t - 2t)^{2/3}} - 1 + 2 \coth^2 t + \frac{1}{4} \frac{(\sinh 2t - 2t)^{4/3}}{h \sinh^4 t} + \\ + \frac{3(\sinh 2t - 2t)^{2/3} (t \coth t - 1)^2}{h^2 \sinh^4 t} + \frac{2}{3} \frac{t \coth t - 1}{(\sinh 2t - 2t)^{2/3} h} - \\ - \frac{2(\sinh 2t - 2t)^{1/3} (t \coth t - 1) \coth t}{h \sinh^2 t}. \end{aligned} \quad (29)$$

³Here we use the same momentum normalization as in the equation (14).

Closing this section we notice that the equation (26) presented here coincides with the equation derived by Krasnitz in the UV limit of the KS theory. The $t \rightarrow \infty$ limit of (26) is the same as the equation (4.30) of [14] with the assignment

$$W_\mu = -\frac{27}{hr^4} K_\mu,$$

and the change to the standard radial variable $r = e^{t/3}$.

5 Supersymmetry and 5d Approach

In this section we compare our findings with the results obtained in the effective five-dimensional models of gauge/gravity correspondence [10] and show that the equations for the graviton and the vector mode are related by a Supersymmetric Quantum Mechanics transformation. This allows us to extend the equation for the vector mode to the baryonic branch.

The authors of [10] systematically study the \mathcal{R} -symmetry invariant sector of fluctuations above the $\mathcal{N} = 2$ backgrounds of the five-dimensional $\mathcal{N} = 8$ gauged supergravity. Those also include the gravity multiplet, i.e. the traceless four-dimensional metric fluctuation and the vector fluctuation, dual to the $U(1)_{\mathcal{R}}$ current.

Although the KS solution truncated to five dimensions would correspond to a more general $\mathcal{N} = 2$ supergravity theory [18], it is nevertheless interesting to compare the results of the two approaches. In fact, in both cases, the unbroken supersymmetry is $\mathcal{N} = 2$ as we deal with the supergravity dual models of $\mathcal{N} = 1$ gauge theories. Therefore the results based on the on-shell supersymmetry can be applicable in both cases. Indeed, we find that SQM transformations that relate the equations for the graviton and the vector mode in the case of the KS background coincide with the supergravity transformations used in [10].

In five-dimensional theories one can use the gauge freedom to recast the background metric into the *kink* form

$$ds_5^2 = dq^2 + e^{2T(q)} \eta_{\mu\nu} dx^\mu dx^\nu. \quad (30)$$

According to a general observation of [12], the traceless graviton fluctuation $h_{\mu\nu}$ in five dimensions satisfies the equation for a scalar minimally coupled to the geometry (30),

$$(\partial_q^2 + 4T' \partial_q - e^{-2T} k^2) h_{\mu\nu} = 0. \quad (31)$$

Using the transformations of the effective $\mathcal{N} = 2$ supergravity of [10] one can transform the graviton $h_{\mu\nu}$ into its superpartner – vector field \hat{B}_μ . As a result, the minimal scalar equation transforms into

$$\left(\partial_q e^{2T} \partial_q - k^2 + 2e^{2T} \frac{\partial^2 T}{\partial q^2} \right) \hat{B}_\mu = 0. \quad (32)$$

Here again k is a 4-momentum. We are going to show that \tilde{A}_μ of (26) and \hat{B}_μ are related by a simple field redefinition.

The approach of [10] uses the superpotential, what can be problematic for the backgrounds from the baryonic branch (except for the KS solution) since the corresponding superpotentials are not known. Therefore there is a concern that the equations obtained for the KS may not be applicable for the outer branch. Nevertheless, we notice that the equation itself is W -independent. This already suggests that it is actually valid for any background of the form (30). Below we will give an argument based on supersymmetry that the equation (32) can be applied to the whole baryonic branch.

Let us first show that the equation (32) is the same as the equation (26) after an appropriate field redefinition. One can think of the metric (30) as an effective metric obtained by truncation of the ten dimensional theory with the metric (4) in the PT form, taken in the Einstein frame,

$$ds_{10}^2 = \left(e^{-6p-x} dt^2 + e^{2A} \eta_{\mu\nu} dx^\mu dx^\nu + g_{\alpha\beta}^{(5)} dy^\alpha dy^\beta \right) e^{-\Phi/2}. \quad (33)$$

The metric (30) is then

$$ds_5^2 = \left(e^{-6p-x} dt^2 + e^{2A} \eta_{\mu\nu} dx^\mu dx^\nu \right) \det^{1/3}(g^{(5)}) e^{-4\Phi/3} = \left(e^{-6p-x} dt^2 + e^{2A} \eta_{\mu\nu} dx^\mu dx^\nu \right) e^{-2p+x} e^{-4\Phi/3}, \quad (34)$$

what gives the following identification for the coordinate q and the function $T(q)$:

$$\frac{d}{dq} = e^{4p+2\Phi/3} \frac{d}{dt}, \quad 2T = 2A - 2p + x - \frac{4}{3} \Phi. \quad (35)$$

Hence the equations for the graviton in ten and five dimensions coincide, because they are just minimal scalar equations.

The equation (32) in the PT notations takes the form

$$\begin{aligned} \partial_t^2 \hat{B}_\mu + (2\dot{p} + \dot{x} + 2\dot{A} - \frac{2}{3} \dot{\Phi}) \partial_t \hat{B}_\mu - k^2 e^{-2A-6p-x} \hat{B}_\mu + \\ + \left((4\dot{p} + \frac{2}{3} \dot{\Phi}) (2\dot{A} - 2\dot{p} + \dot{x} - \frac{4}{3} \dot{\Phi}) + 2\ddot{A} - 2\ddot{p} + \ddot{x} - \frac{4}{3} \ddot{\Phi} \right) \hat{B}_\mu = 0. \end{aligned} \quad (36)$$

To compare this to (26), derived in KS, set $\Phi = 0$. To match the kinetic terms in two equations one should redefine the field $\hat{B}_\mu = e^{2p} \tilde{A}_\mu$. After redefinition one gets

$$\partial_t^2 \tilde{A}_\mu + (6\dot{p} + \dot{x} + 2\dot{A}) \partial_t \tilde{A}_\mu - k^2 e^{-2A-6p-x} \tilde{A}_\mu + \left(2\dot{p} (6\dot{A} + 3\dot{x}) + 2\ddot{A} + \ddot{x} \right) \tilde{A}_\mu = 0, \quad (37)$$

which is precisely the equation (26) for the KS solution (12).

We can further reduce the five-dimensional equations (31) and (32) to one dimension by taking the square of momentum k^2 to be the eigenvalue $-m^2$. This will reduce the supersymmetry algebra to the Supersymmetric Quantum Mechanics with two differential

operators Q_1 and Q_2 that relate the solutions of the two equations (31) and (32). These operators realize the effective transformations of the supersymmetry algebra that was studied in [10]. Indeed, there are operators Q_1 and Q_2 , such that the equations

$$Q_1 Q_2 h_{\mu\nu} = -m^2 h_{\mu\nu} \quad \text{and} \quad Q_2 Q_1 \hat{B}_\mu = -m^2 \hat{B}_\mu \quad (38)$$

coincide with the equation for the graviton (14) and the equation for the vector mode (26) in the form (36). It is easy to show that the operators that satisfy (38) are

$$Q_1 = (\partial_q + 2T') = e^{4p+2\Phi/3} \left(\partial_t + 2\dot{A} - 2\dot{p} + \dot{x} - \frac{4}{3} \dot{\Phi} \right) \quad (39)$$

and

$$Q_2 = e^{2T} \partial_q = e^{2A+2p+x-2\Phi/3} \partial_t. \quad (40)$$

The operator Q_2 is precisely the operator from (73) of [10] that realizes an $\mathcal{N} = 2$ supergravity transformation relating $h_{\mu\nu}$ and \hat{B}_μ .

To get a more conventional representation of the SQM here, one can change the coordinates to $\partial_q = e^{-T} \partial_u$ and bring the equations (31) and (32) to the form (15) by redefining the wave functions $h_{\mu\nu}$ and \hat{B}_μ . Let us define an operator

$$Q = \begin{pmatrix} 0 & \partial_u - W \\ \partial_u + W & 0 \end{pmatrix} \quad (41)$$

with $W = -3T'/2$, that acts on the vector made of redefined wave functions ψ_h and ψ_B . According to the equations (31) and (32) the action of Q^2 is as follows

$$Q^2 \begin{pmatrix} \psi_h \\ \psi_B \end{pmatrix} = -m^2 \begin{pmatrix} \psi_h \\ \psi_B \end{pmatrix}. \quad (42)$$

Therefore Q^2 is analogous to the Hamiltonian of the SQM. Notice, however, that its eigenvalues are m^2 , not m , because Q_1 and Q_2 correspond to the squares of the original supersymmetry transformations, i.e. Q_1, Q_2 are bosonic operators.

We see now that the equation (31) and (32) are related by supersymmetry transformation for any background (30). Since the minimal scalar equation describing the graviton is valid for the whole branch, the superpartner of the graviton (the transverse vector mode) satisfies the “superpartner” equation (32) for any background from the baryonic branch.⁴

We have calculated the spectrum of both equations numerically for the backgrounds along the baryonic branch. Since the equations for the superpartners are significantly different the discrepancy between the masses can be used as an error estimate of the numerical method used in the calculation.

⁴ In general, there is a family of equations like (32) that are related to (31) by a supersymmetry transformation. Indeed, for a given W from (41), any \tilde{W} that satisfies $\tilde{W}^2 + \tilde{W}' = W^2 + W'$ gives rise to such an equation through (42). Nevertheless, the equation (32) is uniquely specified by a requirement that the effective potential V_1 is singular at $t = 0$. This is true because V_1 is singular in the KS case (29) and hence should be singular everywhere on the branch by continuity.

6 Numerical Analysis

In this section we present the results of the numerical studies of bound state spectra for the baryonic branch backgrounds. In our computations we will rely on the shooting technique. The spectrum of the minimal scalar equation (14) in the KS background was also studied numerically in [16, 17, 19] while the analytical approximation was employed in [13].

We start by comparing the KS spectra of the equations for graviton (14) and vector mode (26). Two fluctuations are related by supersymmetry and thus their masses should be the same. The spectrum is presented in the table 1. The eigenvalues match with those obtained by Krasnitz [17] with the WKB approximation. Comparing the numeric values of the masses of the spin-2 and vector particles in the table 1 one could estimate the error of the shooting technique in the KS case to be around 0.1%.

n	1	2	3	4	5	6	7	8	9
Graviton	1.764	4.002	7.143	11.19	16.16	22.03	28.83	36.54	45.16
Vector Mode	1.762	3.999	7.136	11.18	16.12	22.01	28.80	36.50	45.12

Table 1: The spectrum of m^2 for the gravity multiplet

First few (up to ten) values of m^2 in the KS spectrum can be approximated with a good accuracy by a quadratic fit

$$m_n^2 = 0.46 n^2 + 0.86 n + 0.46, \quad n = 1, 2, 3, \dots \quad (43)$$

We present the results of the fit and the masses on the figure 1(a). It is interesting that the fit (43) is close to the spectrum even for small n . The fitting formula (43) is proportional to $(n + n_0)^2$, where n_0 is close to one. This is consistent with the approximation of [13], where the eigenvalues were matched to zeroes of the Bessel functions, ubiquitous in the conical geometry. A similar result was obtained in [20] for the GPPZ [21] flow, where the exact spectrum was proportional to $(n + 1)^2$.

The fit (43) was found by minimizing the sum

$$\sum_{n=1}^N |m_n^2 - (c_2 n^2 + c_1 n + c_0)|^2 \quad (44)$$

for the few first states $N = 5, \dots, 10$. With more points taken into account the least square fit would increase the accuracy of the highest coefficient c_2 by the price of a larger deviation from m_n^2 for small n . We found c_2 to be ~ 0.459 in the KS case. This number is in good agreement with the universal coefficient obtained by Berg, Haack and Mück in [19]. In their normalization the coefficient takes value $(3/4)^{2/3} h(0) c_2 \simeq 0.27$.

Remarkably, the coefficient c_2 does not depend on the details of the effective potential, but rather encodes information about the background geometry, namely, the combination

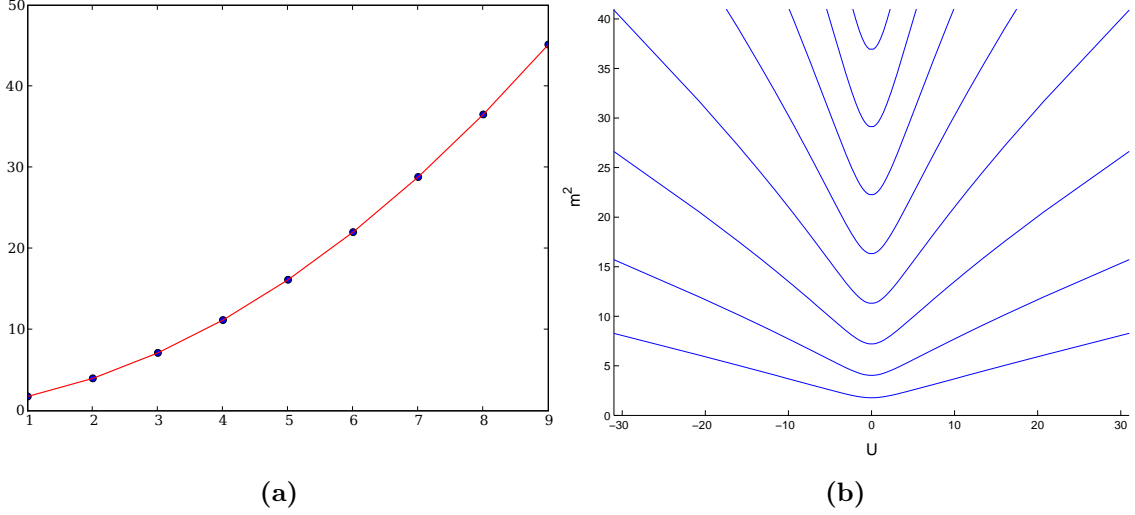


Figure 1: **(a)** Values of m^2 for the graviton multiplet in KS for different quantum numbers n . **(b)** Extension of the spectrum to the baryonic branch parameterized by U .

$g^{00}g_{tt}$, which arises from the Laplace operator in five dimensions. Indeed, the WKB approach, applied in [17], gives

$$\int_0^{t^*} dt \sqrt{-V_2(t)} \Big|_{k^2=-m_n^2} = \frac{3}{4} \pi + (n-1)\pi, \quad (45)$$

where $V_2(t^*) = 0$. In the KS case V_2 is given by (16). For large n , and consequently large m_n , the k^2 -independent term in V_2 can be dropped and we obtain an analytical expression for c_2 in the KS case

$$c_2 = \pi^2 \left[\int_0^\infty dt \frac{\sqrt{h} \sinh t}{(\sinh 2t - 2t)^{1/3}} \right]^{-2} \sim 0.460. \quad (46)$$

Let us choose the coordinate U , introduced in [5], to parameterize the baryonic branch. To estimate the scale of the spectrum for a non-KS background we rewrite the potential (16) in terms of the PT ansatz [8], substituting k^2 for its eigenvalue $-m^2$:

$$V_2(m^2, t) = -\frac{m^2 e^{-2A+x}}{v} + \frac{2a \cosh t}{v} e^{-3g} - \frac{(a \cosh t + 1)^2 + 2a^2 \sinh^2 t}{v^2} e^{-2g}, \quad (47)$$

where $a(t)$ is another function from the PT ansatz [8], $e^{2g} = -1 - a^2 - 2a \cosh t$, and $v = e^{6p+2x}$.

Although we cannot find the spectrum of m^2 analytically, we can estimate how it scales with the parameter U when we are significantly far from the origin of the branch. We start our analysis with the m^2 -independent part of V_2 , which only slightly varies as we increase U . Indeed, its leading UV ($t \rightarrow \infty$) asymptotic is U -independent:

$$V_2(0, t) = \frac{4}{9} - \frac{(5-2t)}{6} U^2 e^{-4t/3} + \dots; \quad (48)$$

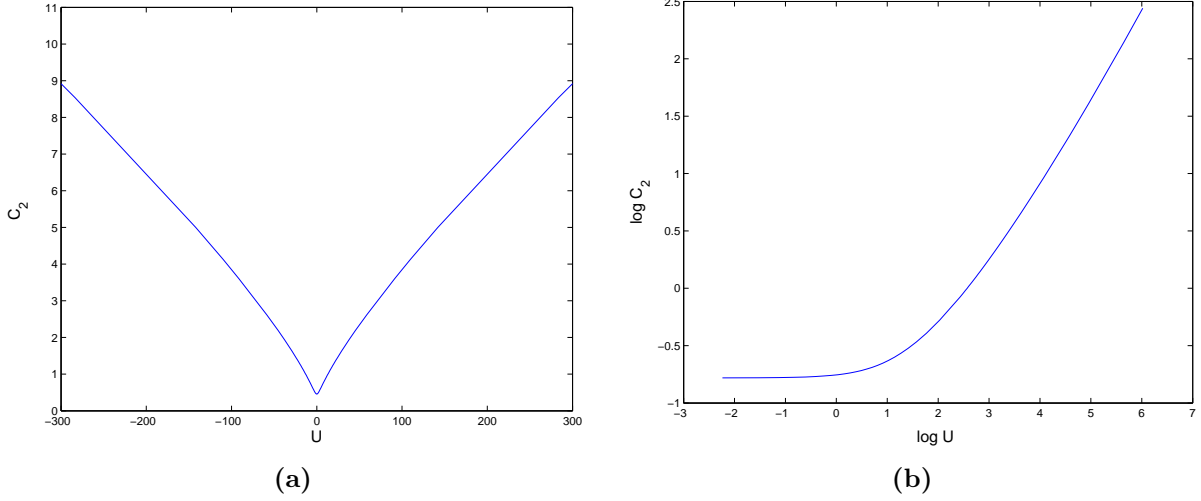


Figure 2: **(a)** c_2 -coefficient as the function of U -parameter. **(b)** $\log c_2$ as the function of $\log U$.

and $V_2(0, t)$ varies within a small range in the IR ($t = 0$):

$$V_2(0, t) = \frac{1}{4} - \frac{3}{5} \xi(1 - \xi) + \mathcal{O}(t^2). \quad (49)$$

Here we remind that $\xi(U) \in (1/6 \dots 5/6)$ [4] is a function of U , which can also be used to parameterize the branch. It varies within the specified limits, and the point $\xi = 1/2$ corresponds to the KS solution. Hence $V_2(0, 0) = 2/5$ for KS and $V_2(0, 0)$ approaches $1/3$ for large U . The $V_2(0, t)$ is monotonic and therefore it can be approximated by a constant in the analysis below.

Unlike $V_2(0, t)$, the mass-dependent component $m^2 e^{-2A+x} v^{-1}$ significantly depends on U . It monotonically changes from a finite value at zero to the zero value at infinity⁵

$$e^{-2A+x} v^{-1} = \frac{2^{1/3} 3}{16} (4t - 1) e^{-2t/3} + \dots \quad (50)$$

In general, the value at zero is a complicated function of U , $\xi(U)$ and $\Phi_0 = \Phi(U, t = 0)$. It can be simplified in the large U range by substituting the limiting value $\xi = 5/6$ and expressing Φ_0 in terms of U and ξ [5]: $e^{\Phi_0} \simeq 2^{3/2} 3^{-1/4} U^{-3/4}$. This gives

$$e^{-2A+x} v^{-1} = \frac{2^{1/3} 3}{2U} \left[1 - e^{2\Phi_0} \left(1 + \frac{2t^2}{9} + \frac{2t^4}{135} + \dots \right) \right]. \quad (51)$$

The normalized solution to the equation (15) exist only if $V_2 < 0$ at the origin, which suggests that m^2 scales at least as U for large U . We can try to be more precise using the semiclassical approximation and express the n -th mass through the integral over the

⁵Here we use the normalization of the warped factor introduced by Krasntitz [17], what results in $e^{-2A} = 2^{1/3} 3 \sqrt{(e^{-2\Phi} - 1)U^{-1}}$.

$V_2 < 0$ region, as we did above in (45). This integral can be roughly approximated as $\sqrt{-V_2(0)}t^* \sim mt^*U^{-1/2}$. The main complication is to estimate t^* . Since $e^{2\Phi_0}$ from (51) is small, the perturbative expansion (51) suggests that t^* increases with U until a point, where (51) is no longer reliable. At the same time the large t asymptotic (50) is U -independent, what suggests that for large U the value of t^* approaches a constant. Therefore we expect $m_n^2 \sim Un^2$ for sufficiently large U .

Numerical studies of the graviton multiplet spectrum on the baryonic branch shows the pattern depicted in the figure 1(b). Calculations confirm that the leading coefficient c_2 grows as U^α , where α approaches 1 for large U (figure 2). As a final touch, we collect in the table 2 the known evidence about the U scaling parameter α for some non-perturbative objects on the baryonic branch.

	SUSY $D5$	Baryonic Condensate	Fundamental String	Glueballs	$D3, \bar{D}3$
α	0	$\alpha < 1$	1/4	1/2	5/4

Table 2: Scale behavior for large U : $T \sim U^\alpha$

7 Discussion

In this work we present the equations describing the bosonic degrees of freedom of the gravity multiplet for the KS and the baryonic branch backgrounds. The equations were derived by a linearization of the ten-dimensional type IIB supergravity equations. The traceless graviton from the gravity multiplet satisfies the equation for a scalar minimally coupled to the background (11). The vector mode of the gravity multiplet dual to the $U(1)_{\mathcal{R}}$ current satisfies the equation (26) in the KS background. Its generalization to the baryonic branch (36) is found by matching it to the equation (32) derived in the five-dimensional approach in [10]. This result is supported by the supersymmetry transformation that relates the wave functions of these fluctuations.

The mass spectrum of the gravity multiplet for the KS background can be found in the table 1. This spectrum can be approximated with a good accuracy by a simple quadratic formula (43), which is approximately

$$m_n^2 \simeq 0.46(n+1)^2. \quad (52)$$

This simple complete square form does not hold along the baryonic branch, although the spectrum can be well approximated by a general quadratic formula $c_2n^2 + c_1n + c_0$.

In this work we did not study the spectrum of the anomaly multiplet S of (1), which contains the fluctuation of the trace of the metric h_μ^μ and the longitudinal part of the vector fluctuation \tilde{a} (5). The main complication is that these fluctuations do not decouple from the other supergravity fields. Recently the fluctuation of the metric h_μ^μ was considered as a part of 7-particle system by Berg, Haack and Mück in [15, 19]. They found the

resulting spectrum of the system, but the individual mass towers were not identified with the glueballs.

Based on the similarities between the spectrum of the gravity multiplet in the KS (52) and GPPZ backgrounds, where $m_n^2 = 4L^{-2}(n+1)^2$, one can assume that some features of the spectra for certain glueballs do not crucially depend on the details of the background. Based on the exact result of the GPPZ case calculation for the mass spectrum of the anomaly multiplet S , $m_n^2 = 4L^{-2}(n+1)(n+2)$ [10, 20, 22], one can guess the answer for the KS case. In the units of BHM the approximate formula reads

$$m_n^2 \simeq 0.27(n+1)(n+2), \quad n = 1, 2, 3, \dots$$

This is in fact close to the lightest of the seven towers of BHM, given by the empirical formula

$$0.271n^2 + 0.774n + 0.562.$$

It would be interesting to confirm the matching between the trace of the metric and the lowest tower of the 7-particle system with a more rigorous approach.

We find it intriguing that the spectrum of [19] contains only two states that look degenerate. Given that we are dealing with massive states of the $\mathcal{N} = 1$ system we would expect all the states to be degenerate. This might signify that the numerical method used in [19] alter the mass degeneracy because of a numerical error. Another possibility is that the superpartners of the glueballs in question are not the part of the 7-particle system and can not be captured by the fluctuations of the PT ansatz. The clarification of the magnitude of the numerical error is also important to check another finding of [19] – the significant deviation of the spectrum from the quadratic behavior for few lowest values of the quantum number n .

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